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Plasma stopping power including subleading order

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Abstract
Dimensional continuation is employed to compute the energy loss rate for a non-relativistic particle moving through a highly ionized plasma. No restriction is made on the charge, mass, or speed of this particle, but it is assumed that the plasma is not strongly coupled in that the dimensionless plasma coupling parameter $g = e^2\kappa_D/4\pi T$ is small, where κ_D is the Debye wave number. To leading order in this coupling, dE/dx is of the generic form $g^2 \ln[g^2 C]$. The prefactor of the logarithm is well known. We compute the constant C under the logarithm exactly. Our result differs from approximations given in the literature, with differences in the range of about 20% for cases relevant to inertial confinement fusion experiments.

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(Some figures in this article are in colour only in the electronic version)

1. dE/dx and the Coulomb log

The stopping power of plasma component b for projectile p is of the form

$$\frac{\mathrm{d}E_b}{\mathrm{d}x} = \frac{e_p^2\, e_b^2}{4\pi}\frac{n_b}{m_b\, v_p^2}\ln\Lambda_b = \frac{e_p^2}{4\pi}\frac{\kappa_b^2}{\beta_b m_b v_p^2}\ln\Lambda_b, \tag{1}$$

where the Coulomb logarithm $\ln\Lambda_b$ involves a ratio of short- and long-distance length scales. To compute $\ln\Lambda_b$, we employ the method of *dimensional continuation* [1]. To introduce this method, we consider the Coulomb potential $\phi_\nu(r)$ of a point source in ν spatial dimensions: $\phi_\nu(r) \sim 1/r^{\nu-2}$. Clearly the long- and short-distance behaviour of ϕ_ν depends on the spatial dimensionality ν. In high ν, short-distance (hard) interactions are accentuated, while in low ν the large-distance (soft) physics predominates.

For Coulomb interactions, $\nu = 3$ is special in that neither hard nor soft processes are dominant. For $\nu < 3$, the soft physics is predominant, and for $\nu > 3$ the hard processes are dominant. The energy loss for $\nu > 3$, $\mathrm{d}E^{\mathrm{B}}/\mathrm{d}x$ is obtained from the Boltzmann (B) equation,

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and it contains a pole $(\nu - 3)^{-1}$ that reflects an infrared divergence in the scattering process when $\nu \to 3^+$. The energy loss for $\nu < 3$, $\mathrm{d}E^{\mathrm{LB}}/\mathrm{d}x$ is obtained from the Lenard–Balescu (LB) kinetic equation, and it contains a pole $(3 - \nu)^{-1}$ that reflects an ultraviolet divergence when $\nu \to 3^-$. The stopping power to subleading order is therefore

$$\frac{\mathrm{d}E}{\mathrm{d}x} = \lim_{\nu\to 3}\left(\frac{\mathrm{d}E^{\mathrm{LB}}}{\mathrm{d}x} + \frac{\mathrm{d}E^{\mathrm{B}}}{\mathrm{d}x}\right), \tag{2}$$

and it is completely finite. Hence the two poles must cancel. The dependence of the residues of the poles on ν brings in a logarithm of the ratio of the relevant short- and long-distance length scales, which is precisely the Coulomb logarithm. The method of dimensional continuation expressed in equation (2) is somewhat subtle, especially the fact that adding the $\nu < 3$ and $\nu > 3$ pieces yields precisely the leading and sub-leading behaviour. We refer the reader to the simple example in appendix A of [1], which clearly illustrates the validity of this technique.

2. Collective excitations: Lenard–Balescu equation for $\nu < 3$

The soft physics is described to leading order in the plasma density by

$$\frac{\partial}{\partial t} f_a(\mathbf{p}_a) = -\sum\nolimits_b \frac{\partial}{\partial \mathbf{p}_a}\cdot \mathbf{J}_{ab}, \tag{3}$$

which is the Lenard–Balescu kinetic equation for plasma species a and b, where

$$\mathbf{J}_{ab} = e_a^2 e_b^2 \int \frac{\mathrm{d}^\nu \mathbf{k}}{(2\pi)^\nu}\frac{\mathbf{k}}{(\mathbf{k}^2)^2}\frac{\pi}{|\epsilon(\mathbf{k}, \mathbf{v}_a\cdot\mathbf{k})|^2} \times \int \frac{\mathrm{d}^\nu \mathbf{p}_b}{(2\pi\hbar)^\nu}\delta(\mathbf{k}\cdot(\mathbf{v}_a - \mathbf{v}_b))\mathbf{k}\cdot\left[\frac{\partial}{\partial \mathbf{p}_b} - \frac{\partial}{\partial \mathbf{p}_a}\right] f_a(\mathbf{p}_a) f_b(\mathbf{p}_b), \tag{4}$$

with $\mathbf{v}_a = \mathbf{p}_a/m_a$ and $\mathbf{v}_b = \mathbf{p}_b/m_b$. The collective behaviour of the plasma enters through its dielectric function

$$\epsilon(\mathbf{k},\omega) = 1 + \sum\nolimits_c \frac{e_c^2}{k^2}\int \frac{\mathrm{d}^\nu \mathbf{p}_c}{(2\pi\hbar)^\nu}\frac{1}{\omega - \mathbf{k}\cdot\mathbf{v}_c + \mathrm{i}\eta}\mathbf{k}\cdot\frac{\partial}{\partial \mathbf{p}_c} f_c(\mathbf{p}_c), \qquad \eta \to 0^+. \tag{5}$$

The rate of kinetic energy loss of species a to species b is given by

$$\frac{\mathrm{d}\mathcal{E}_{ab}^{\mathrm{LB}}}{\mathrm{d}t} = \int \frac{\mathrm{d}^\nu \mathbf{p}_a}{(2\pi\hbar)^\nu}\frac{p_a^2}{2m_a}\frac{\partial}{\partial \mathbf{p}_a}\cdot \mathbf{J}_{ab}. \tag{6}$$

We evaluate this for the case in which species a is a single projectile of mass m_p and velocity $\mathbf{v}_p$, $f_a(\mathbf{p}_a) = (2\pi\hbar)^\nu \delta^{(\nu)}(\mathbf{p}_a - m_p\mathbf{v}_p)$ and the distribution function $f_b(\mathbf{p}_b)$ for plasma species b is Maxwell–Boltzmann at temperature $T_b = 1/\beta_b$. With $\mathrm{d}x = v_p\,\mathrm{d}t$,

$$\begin{aligned}\frac{\mathrm{d}E_b^{\mathrm{LB}}}{\mathrm{d}x} &= \frac{e_p^2}{4\pi}\frac{1}{\beta_b m_p v_p^2}\frac{\Omega_{\nu-2}}{2\pi}\left(\frac{K}{2\pi}\right)^{\nu-3}\frac{1}{3-\nu}\int_0^1 \mathrm{d}u\,(1-u)^{(\nu-3)/2}\rho_b(v_p u^{1/2})\left[\beta_b M_{pb} v_p^2 - \frac{1}{u}\right] \\ &\quad + \frac{e_p^2}{4\pi}\frac{\mathrm{i}}{2\pi}\int_{-1}^{+1} d\cos\theta\cos\theta\,\frac{\rho_b(v_p\cos\theta)}{\sum_c \rho_c(v_p\cos\theta)}F(v_p\cos\theta)\ln\!\left(\frac{F(v_p\cos\theta)}{K^2}\right) \\ &\quad - \frac{e_p^2}{4\pi}\frac{\mathrm{i}}{2\pi}\frac{1}{\beta_b m_p v_p^2}\frac{\rho_b(v_p)}{\sum_c\rho_c(v_p)}\left[F(v_p)\ln\left(\frac{F(v_p)}{K^2}\right) - F^*(v_p)\ln\left(\frac{F^*(v_p)}{K^2}\right)\right],\end{aligned} \tag{7}$$

where $M_{pb} = m_p + m_b$ is the total mass, $F(v) = k^2[\epsilon(\mathbf{k}, kv) - 1]$ and $\rho_b(v) = \kappa_b^2\sqrt{\beta_b m_b/2\pi}\, v\exp(-\beta_b m_b v^2/2)$. Here Ω_ν is the area of a unit sphere in ν dimensions and K is an arbitrary wave number whose dependence cancels in the limit (2).

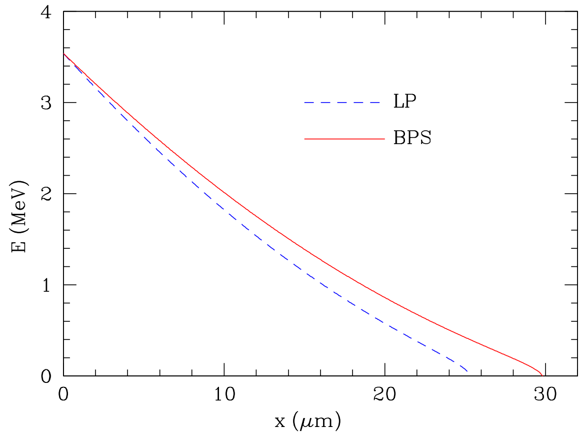

Figure 1. The energy $E(x)$ (in MeV) of an α particle with initial energy $E_0 = 3.54$ MeV versus the distance x (in μm) travelled through an equal molal DT plasma. Note that $E(x)$ is obtained by inverting $x = \int_{E_0}^{E} \mathrm{d}E(\mathrm{d}E/\mathrm{d}x)^{-1}$, where the stopping power dE/dx has been expressed as a function of energy. The plasma temperature is $T = 3$ keV and the electron number density is $n_e = 10^{25}$ cm^{-3}. The plasma coupling is small, $g = 0.011$, and so our calculation (BPS) is essentially exact. Our result is shown by the solid curve. The work of Li and Petrasso [2] is often used in laser fusion simulations. Their result (LP) is shown by the dashed line. Note that the difference in the total ranges between our result and that of Li and Petrasso of about 5 μm is a little larger than 20%.

3. Hard collisions: Boltzmann equation for $\nu > 3$

Hard collisions in the plasma density are described by the Boltzmann equation, giving

$$\frac{\mathrm{d}E_b^{\mathrm{B}}}{\mathrm{d}x} = \frac{1}{v_p} \int \frac{\mathrm{d}^\nu \mathbf{p}_b}{(2\pi\hbar)^\nu} f_b(\mathbf{p}_b) v_{pb} \int \mathrm{d}\sigma_{pb}\, \frac{1}{2} m_p \left[v_p'^2 - v_p^2 \right], \tag{8}$$

where $v_{pb} = |\mathbf{v}_p - \mathbf{v}_b|$, and $\mathrm{d}\sigma_{pb}$ is the full quantum-mechanical differential cross section for scattering of the projectile (p) from the initial velocity $\mathbf{v}_p = \mathbf{p}_p/m_p$ to the final velocity $\mathbf{v}'_p$ off a plasma particle (b). Straightforward kinematical manipulations exploiting the axial symmetry of the scattering produce the form

$$\frac{\mathrm{d}E_b^{\mathrm{B}}}{\mathrm{d}x} = \frac{1}{v_p} \int \frac{\mathrm{d}^\nu \mathbf{p}_b}{(2\pi\hbar)^\nu} f_b(\mathbf{p}_b) \frac{\mathbf{P}\cdot\mathbf{p}}{2p^2 M_{pb}} v_{pb} \int \mathrm{d}\sigma_{pb}\, q^2, \tag{9}$$

in which $\mathbf{P}$ is the total momentum of the centre of mass, $\mathbf{p}$ is the relative momentum in the centre of mass and $\mathbf{q}$ is the momentum transfer.

The *classical* cross section in ν dimensions is $\mathrm{d}\sigma_{pb}^{\mathrm{C}} = \Omega_{\nu-2}\, B^{\nu-2}\, \mathrm{d}B$, where B is the classical impact parameter. Some calculation gives

$$\int \mathrm{d}\sigma_{pb}^{\mathrm{C}} q^2 = \frac{\Omega_{\nu-2}}{2\pi} \frac{m_{pb}^2}{p^2} \frac{(e_p e_b)^2}{2\pi} \left[\frac{p^{2(\nu-3)}}{\nu-3} - \ln\left(\frac{e_p e_b m_{pb}}{4} \right) - \gamma \right], \tag{10}$$

with $m_{pb} = m_p m_b / M_{pb}$ being the reduced mass. Placing the result (10) in equation (9) yields

$$\begin{aligned} \frac{\mathrm{d}E_{b\,\mathrm{C}}^{\mathrm{B}}}{\mathrm{d}x} = \frac{e_p^2}{4\pi} \frac{1}{\beta_b m_p v_p^2} \int_0^1 \mathrm{d}u\, \rho_b(v_p\sqrt{u}) \Bigg\{ \Bigg[\frac{\Omega_{\nu-2}}{2\pi} \frac{1}{\nu-3} (1-u)^{(\nu-3)/2} \\ + 2 - 2\gamma - \ln\left(\frac{e_p e_b \beta_b m_b}{2m_{pb}} \frac{u}{1-u} \right) \Bigg] \left(\beta_b M_{pb} v_p^2 - \frac{1}{u} \right) + \frac{2}{u} \Bigg\}. \end{aligned} \tag{11}$$

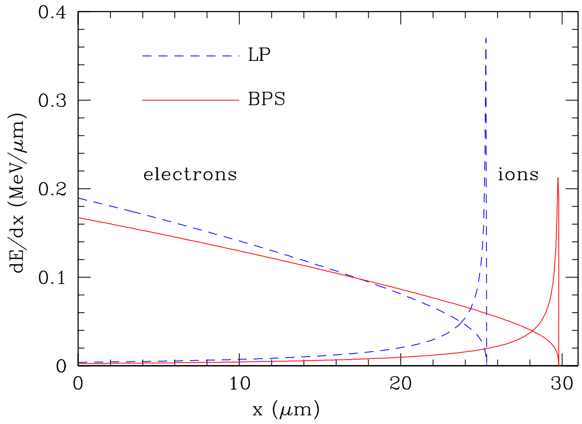

Figure 2. The α particle dE(x)/dx (in Mev/μm) versus x (in μm) split into separate ion (spiked curves) and electron components (softly decreasing curves). The area under each curve gives the corresponding energy partition into electrons and ions. For our results (BPS), the total energy deposited into electrons is $E_e = 3.16$ MeV and into ions is $E_I = 0.38$ MeV, while LP gives $E_e^{\rm LP} = 3.11$ MeV and $E_I^{\rm LP} = 0.43$ MeV. These energies sum to the initial α particle energy of $E_0 = 3.54$ MeV. Note that BPS has a longer α particle range and deposits less energy into ions than LP. Both observations would tend to make fusion more difficult to achieve for BPS than for LP.

Making the decomposition $\int \mathrm{d}\sigma_{pb} q^2 = \int \mathrm{d}\sigma_{pb}^{\rm C} q^2 + \int \left(\mathrm{d}\sigma_{pb} - \mathrm{d}\sigma_{pb}^{\rm C}\right) q^2$ expresses $\mathrm{d}E_b^{\rm B}/\mathrm{d}x = \mathrm{d}E_{b\,\rm C}^{\rm B}/\mathrm{d}x + \mathrm{d}E_{b\,\rm Q}^{\rm B}/\mathrm{d}x$, where $\mathrm{d}E_{b\,\rm Q}^{\rm B}/\mathrm{d}x$ is the *quantum* correction to equation (11). The integral $\int \left(\mathrm{d}\sigma_{pb} - \mathrm{d}\sigma_{pb}^{\rm C}\right) q^2$ is most easily evaluated by first calculating $\int (\mathrm{d}\sigma_{pb} - \mathrm{d}\sigma_{pb}^{\rm B}) q^2$, where $\mathrm{d}\sigma_{pb}^{\rm B}$ is the Born approximation to $\mathrm{d}\sigma_{pb}$, and then subtracting the contribution $\int \left(\mathrm{d}\sigma_{pb}^{\rm C} - \mathrm{d}\sigma_{pb}^{\rm B}\right) q^2$. Inserting the correction $v_{pb} \int \left(\mathrm{d}\sigma_{pb} - \mathrm{d}\sigma_{pb}^{\rm C}\right) q^2$ into equation (9) yields

$$\frac{\mathrm{d}E_{b\,\rm Q}^{\rm B}}{\mathrm{d}t} = \frac{e_p^2 \kappa_b^2}{4\pi} \frac{2 v_p}{\sqrt{2\pi\alpha_b}} e^{-\alpha_b/2} \int_0^\infty \mathrm{d}u \left\{ e^{-\alpha_b u^2/2} [\ln(\eta_b/u) - \mathrm{Re}\, \psi(1 + \mathrm{i}\eta_b/u)] \right. \\ \left. \times \left[\frac{M_{pb}}{m_p\, u} \left(\cosh \alpha_b u - \frac{\sinh \alpha_b u}{\alpha_b u} \right) - \frac{m_b}{m_p} \sinh \alpha_b u \right] \right\}, \tag{12}$$

where $\psi(z) = \mathrm{d} \ln \Gamma(z)/\mathrm{d}z$, Re denotes the real part, $\alpha_b \equiv \beta_b m_b v_p^2$ and $\eta_b \equiv e_p e_b / 4\pi\hbar v_p$.

4. Results

The total stopping power is the sum of the contributions from large-distance collective excitations $\mathrm{d}E^{\rm LB}/\mathrm{d}x$ and from short-distance hard collisions $\mathrm{d}E^{\rm B}/\mathrm{d}x$, that is, the sum over species b of equations (7), (11) and (12). The poles at $\nu = 3$ and the $\ln K$ terms cancel. Our result for $\mathrm{d}E/\mathrm{d}x$ is generically of the form $n\,(\ln n + C)$ in the plasma density n, and it is accurate to all orders in the quantum parameter η_b. Figures 1 and 2 illustrate our result with an example that is relevant to the DT plasmas in laser fusion capsules.

References

[1] Brown L S, Preston D L and Singleton R L Jr 2005 *Phys. Rep.* **410** 237 (*Preprint* physics/0501084)
[2] Li C-K and Petrasso R D 1993 *Phys. Rev. Lett.* **70** 3059